Stabilizer models of gapped quantum phases of matter

Tyler Ellison
(he/him)
PERIMETER INSTITUTE

Quantum **5**, 612
PRX Quantum **3**, 010353
PRX Quantum **3**, 030326
Quantum **7**, 1137
arXiv:2405.02390
arXiv:2411.04993

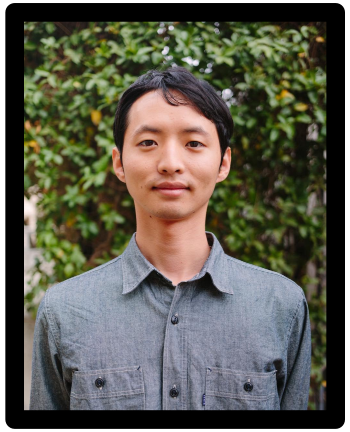
Collaborators

Yu-An Chen

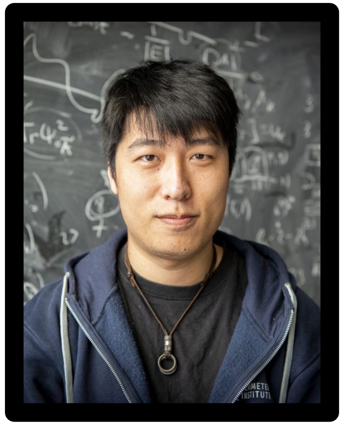
Meng Cheng

Arpit Dua

Tim Hsieh



Kohtaro Kato



Zi-Wen Liu

Julio Magdalena de la Fuente

Nat Tantivasadakarn

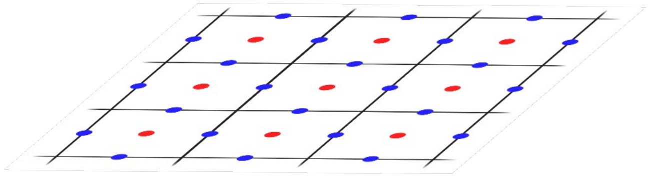
Wilbur Shirley

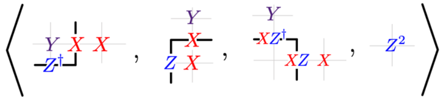
Dom Williamson

Motivations

Which quantum phases can be described within the stabilizer formalism?

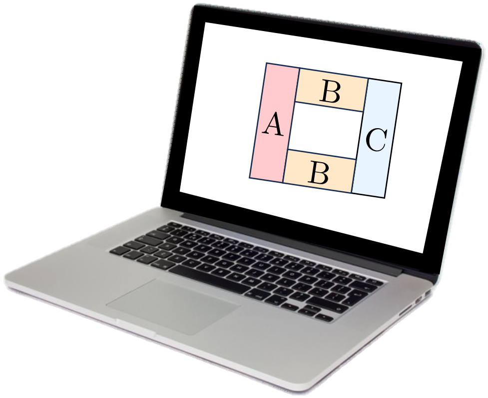
New quantum error-correcting codes





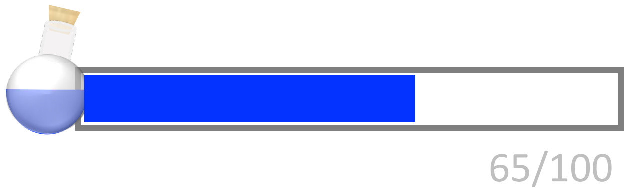
- Improve robustness to errors
- Reduce overheads
- Address limitations of hardware

Computation and simulation-friendly models



Ex: topological entanglement negativity

Assess quantum complexity of the phase



- Identify obstructions to capturing universal properties with stabilizer states

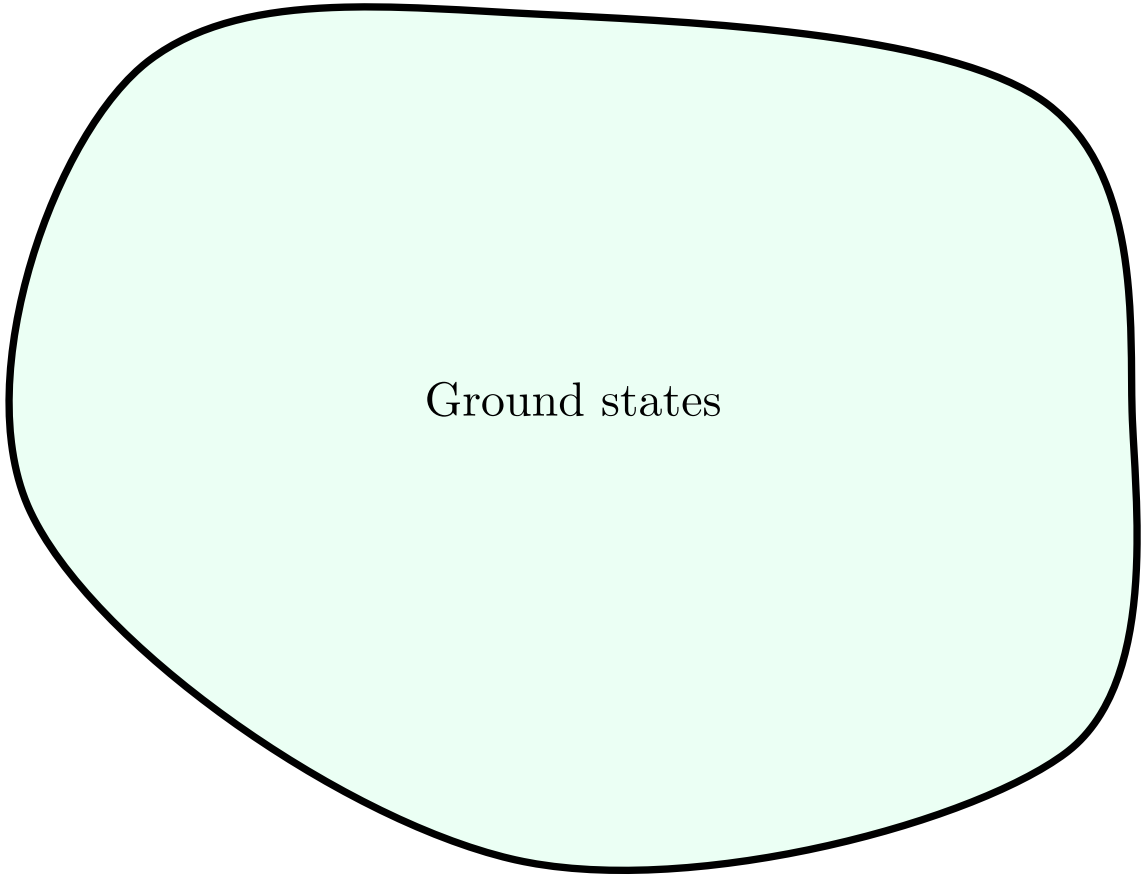
Outline

- Gapped quantum phases of matter
- Anyon theories
- No-go theorems for the stabilizer formalism
- Go-around theorems:
  - composite-dimensional qudits, continuous variables, holographic approach, mixed states
- Open questions

Gapped quantum phases of matter

<u>Which states?</u>

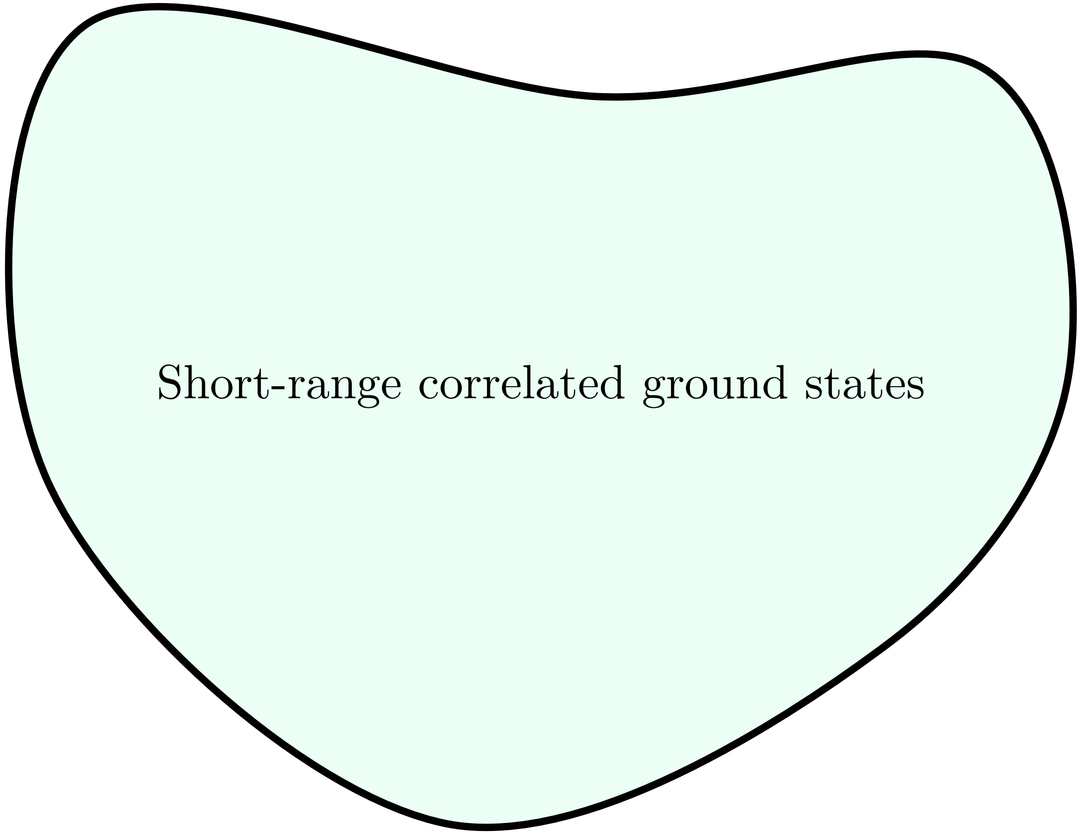
- Ground states of Hamiltonians:
    1. Geometrically local terms
    2. Gapped (in thermodynamic limit)



Gapped quantum phases of matter

<u>Which states?</u>

- Ground states of Hamiltonians:
    1. Geometrically local terms
    2. Gapped (in thermodynamic limit)
- Short-range correlations* (rules out GHZ)



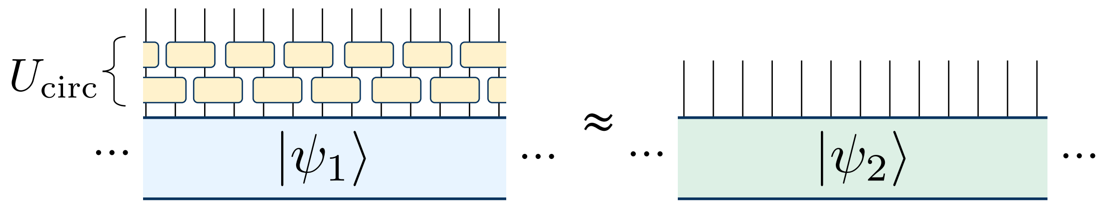
* $\langle\psi|\mathcal{O}_i\mathcal{O}_j|\psi\rangle - \langle\psi|\mathcal{O}_i|\psi\rangle\langle\psi|\mathcal{O}_j|\psi\rangle \to 0$, for all local $\mathcal{O}_i, \mathcal{O}_j$

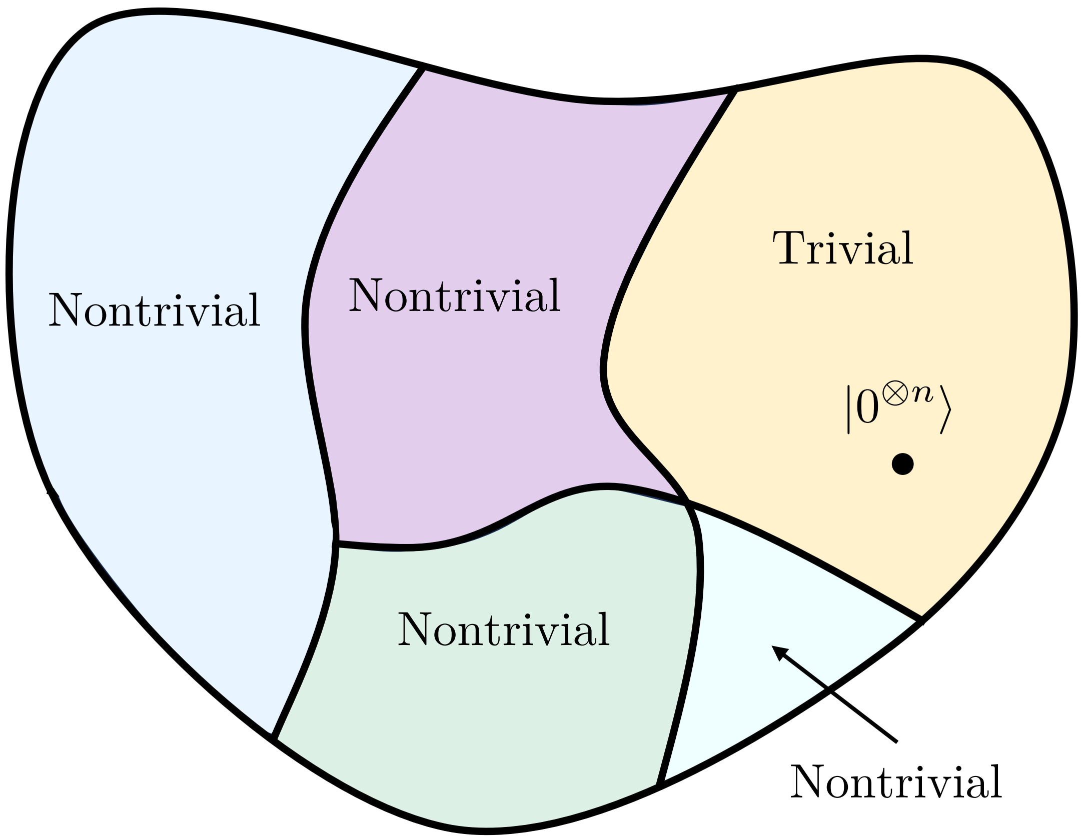
Gapped quantum phases of matter

<u>Equivalence relation:</u>

$|\psi_1\rangle \sim |\psi_2\rangle$ if there exists a constant-depth*

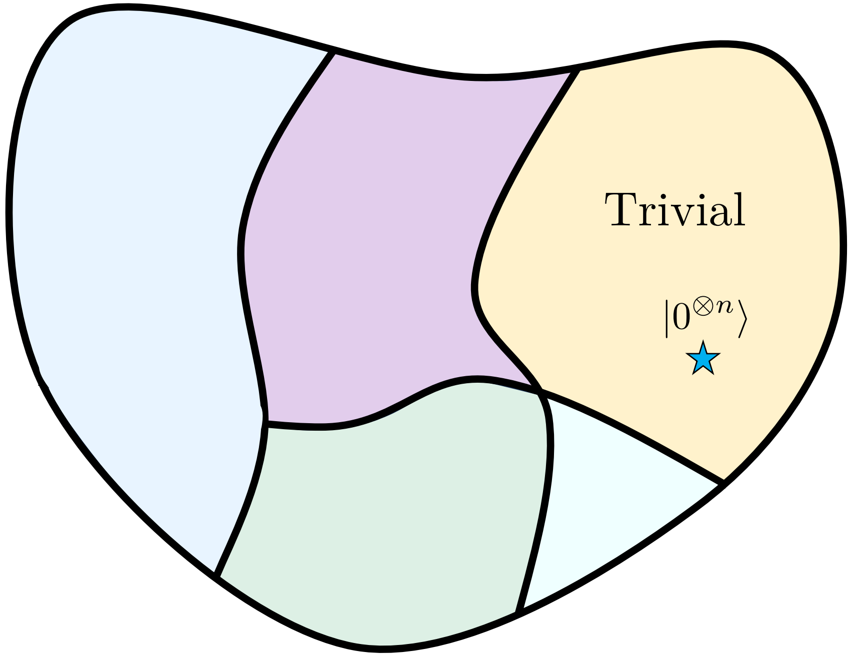
circuit such that $U|\psi_1\rangle = |\psi_2\rangle$





*More generally, polylog-depth

Main question



Which phases admit a stabilizer state? Which phases have universal properties that can be captured by the stabilizer formalism?

Gapped quantum phases of matter in 2D

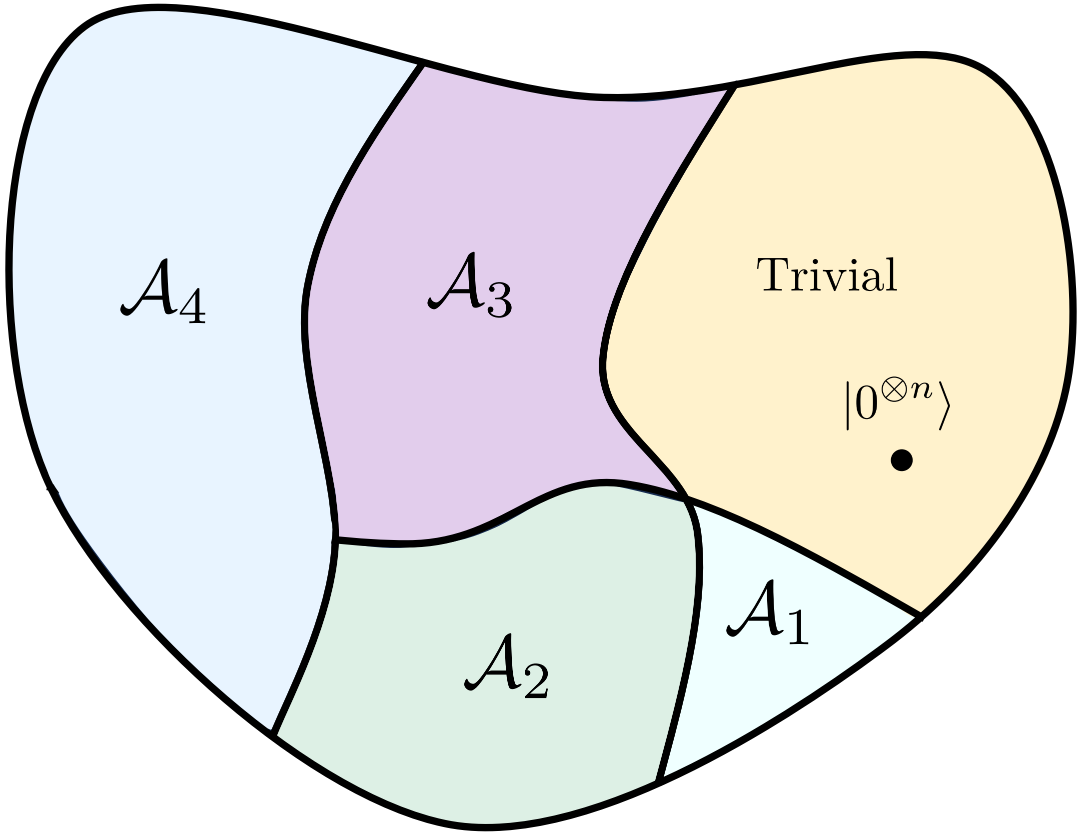
Classification:*

- Anyon theories!

  (modular tensor categories)

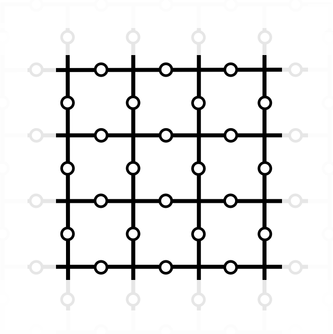
Anyons:

$$\left\{ \begin{array}{c} \text{Local} \\ \text{excitations} \end{array} \right\} \Big/ \left\{ \begin{array}{c} \text{Local excitations} \\ \text{created by local operators} \end{array} \right\}$$

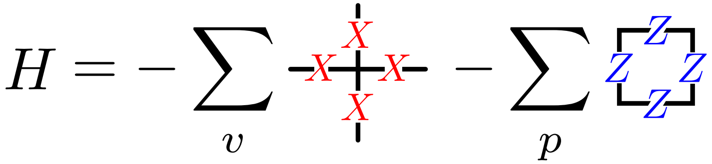


*Up to stacking with E_8 states

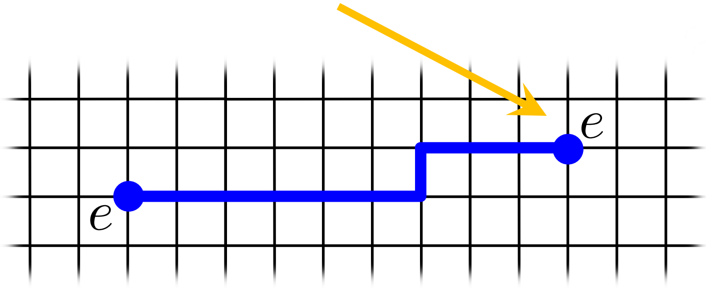
Surface code anyons



∘ = qubit,



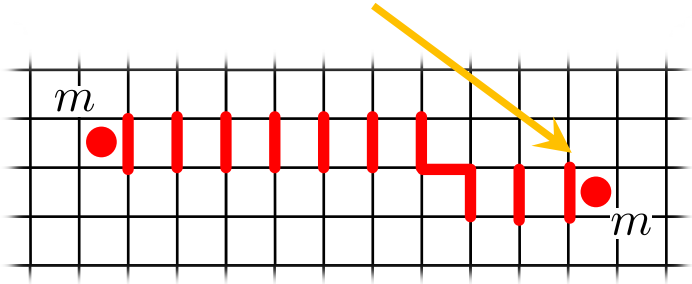
Violates vertex term



= Z

(Gauge charge)

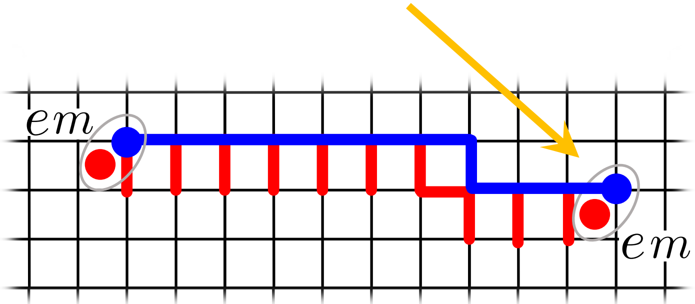
Violates plaquette term



= X

(Gauge flux)

Violates both vertex and plaquette terms

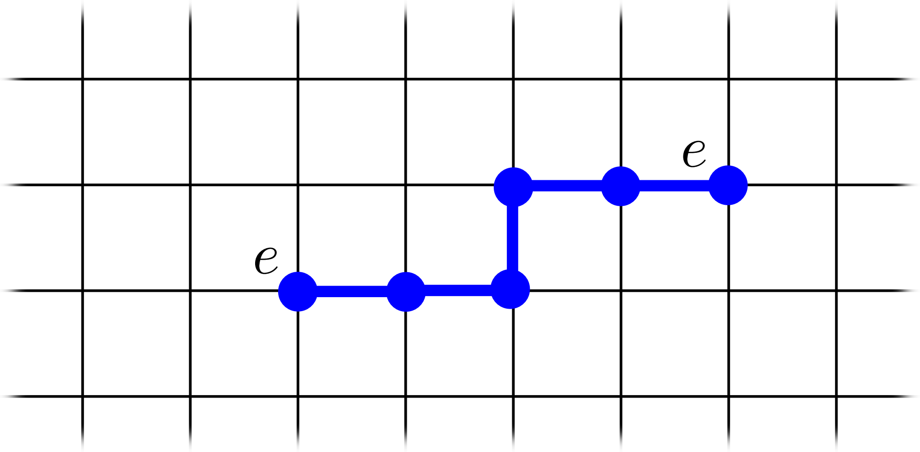


= Z = X

(Dyon)

Surface code anyons properties

Fusion



$$e \times e = 1$$

Deterministic fusion → "Abelian anyons"

~~Non-deterministic fusion → "non-Abelian anyons"~~ → No stabilizer models

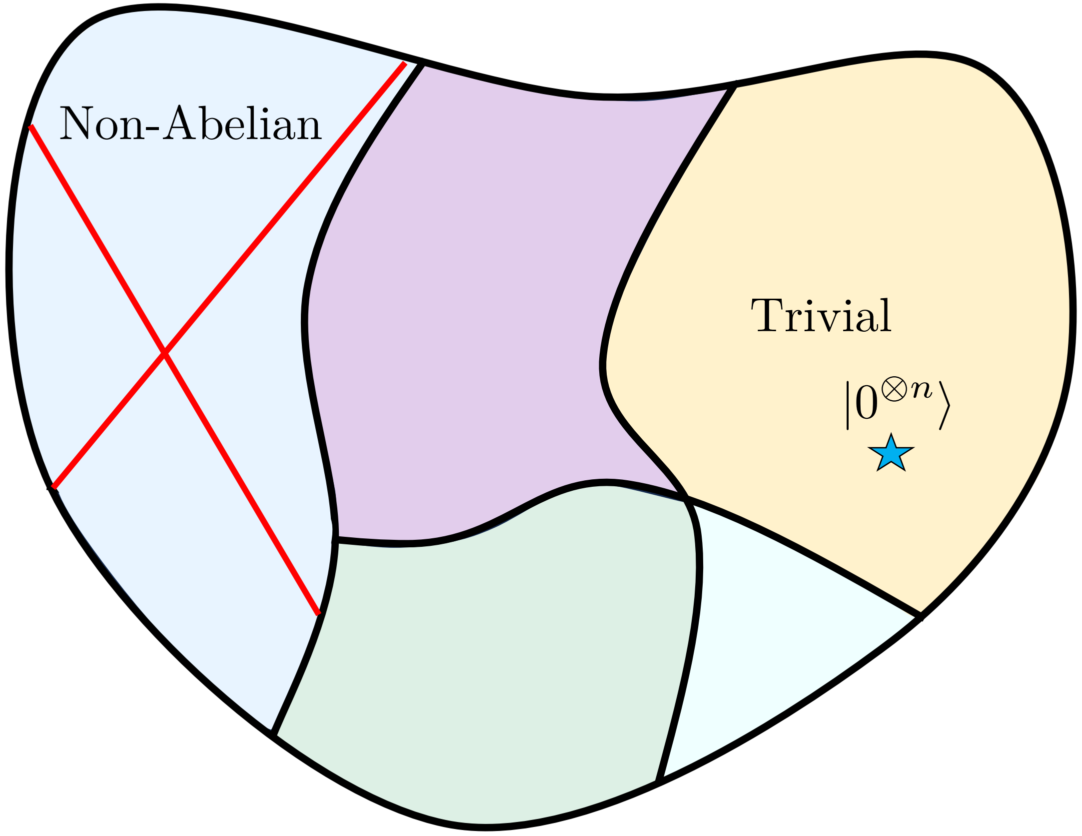
Which phases admit a stabilizer state?

Trivial phase:

Any computational basis state ☑

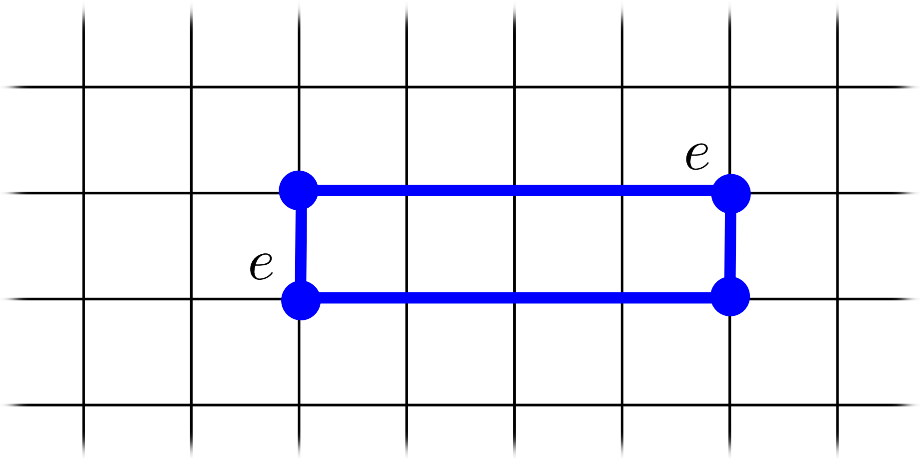
Phases with non-Abelian anyons:

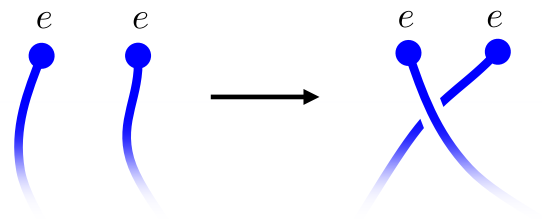
Cannot reproduce non-deterministic fusion ☒



Surface code anyons properties

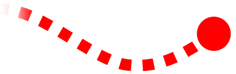
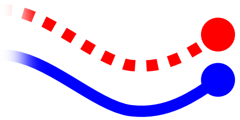
Exchange





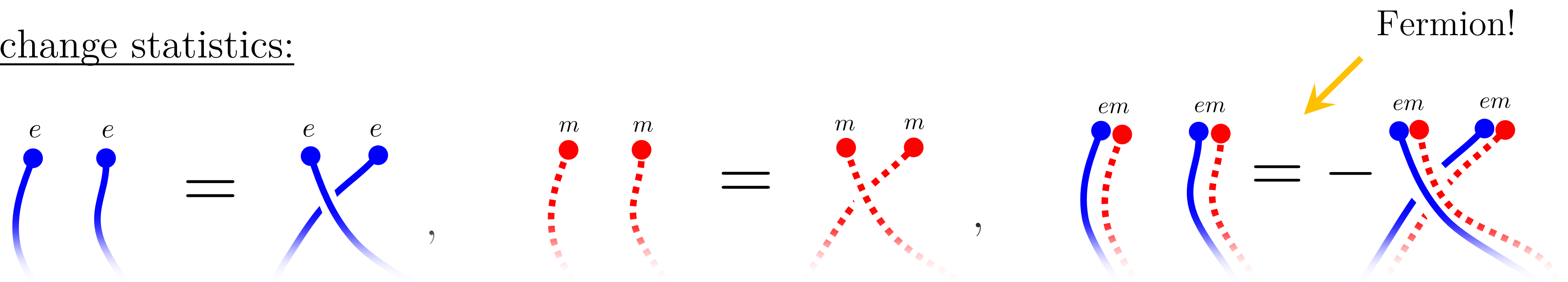
Surface code anyon theory

Anyon types: $\{1, e, m, em\}$

$e =$  , $m =$  , $em =$ 

Fusion rules: $e \times e = 1$, $m \times m = 1$, $e \times m = em$ ...

Exchange statistics:



(Braiding relations are determined by the exchange statistics)

Abelian anyon theories

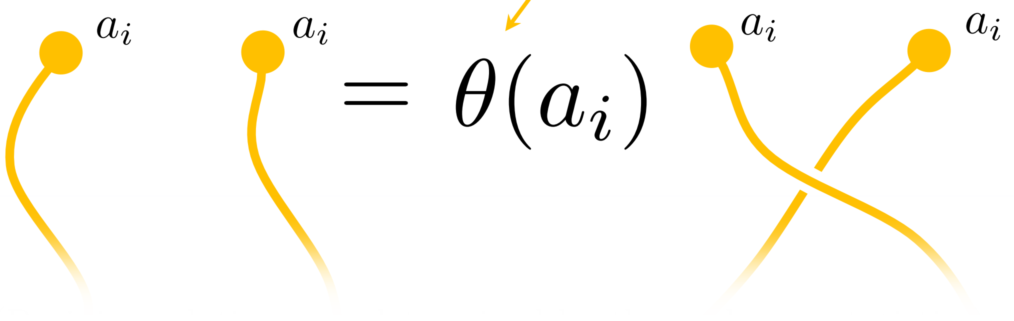
Anyon types: $\{1, a_1, a_2, \ldots, a_n\}$

Fusion rules: $a_i \times a_j = a_k, \quad a_i \times 1 = a_i$

Many Abelian anyon theories beyond anyon theories of toric codes!

Exchange statistics:

Satisfies certain consistency conditions



(Braiding relations are determined by the exchange statistics)

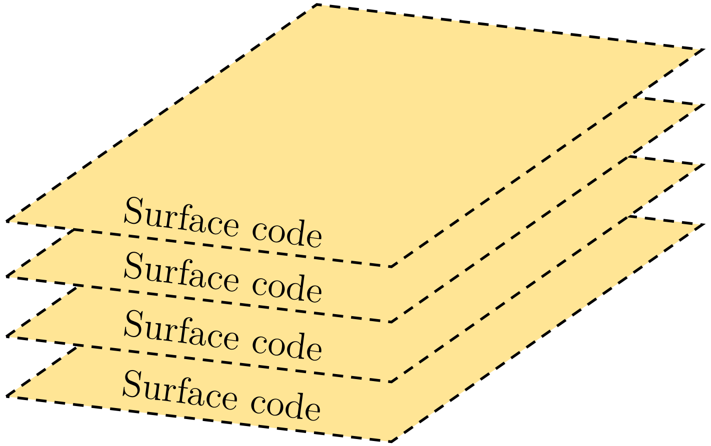
Classification of 2D topological Pauli stabilizer codes

Do Pauli stabilizer c… …eories that are more exotic th… …code?

Yes!

No-go theorem: every translation invariant Pauli stabilizer code in 2D is constant-depth equivalent to layers of surface codes

- Shown rigorously for prime-dimensional qudits



On prime-dimensional qudits

What about composite-dimensional qudits, continuous variables, boundaries of 3D systems, mixed states?!

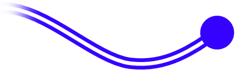
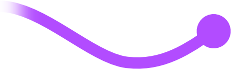
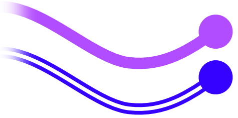
H. Bombin, G. Duclos-Cianci, D. Poulin 2011; H. Bombin 2011; J. Haah 2018

Composite-dimensional qudits

PRX Quantum **3**, 010353

Double semion anyon theory

Anyon types: $\{1, b, s, \bar{s}\}$

$b =$  , $s =$  , $\bar{s} =$ 

Distinct from the toric code or a stack of toric codes!

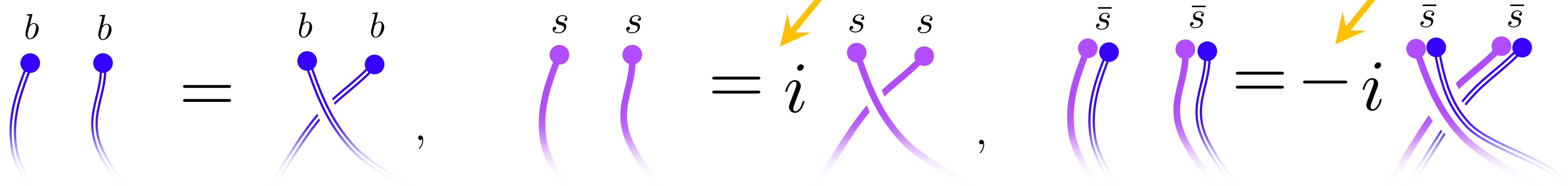
Fusion rules:

$$b \times b = 1 \;, \quad s \times s = 1 \;, \quad \bar{s} \times \bar{s} = 1 \;, \quad b \times s = \bar{s} \;, \; \ldots$$

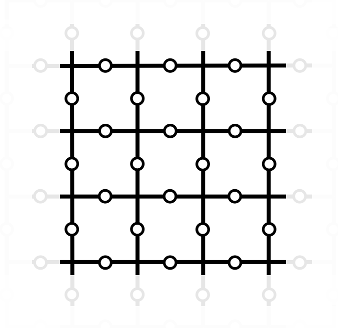
Exchange statistics:

Semion!

Anti-semion!



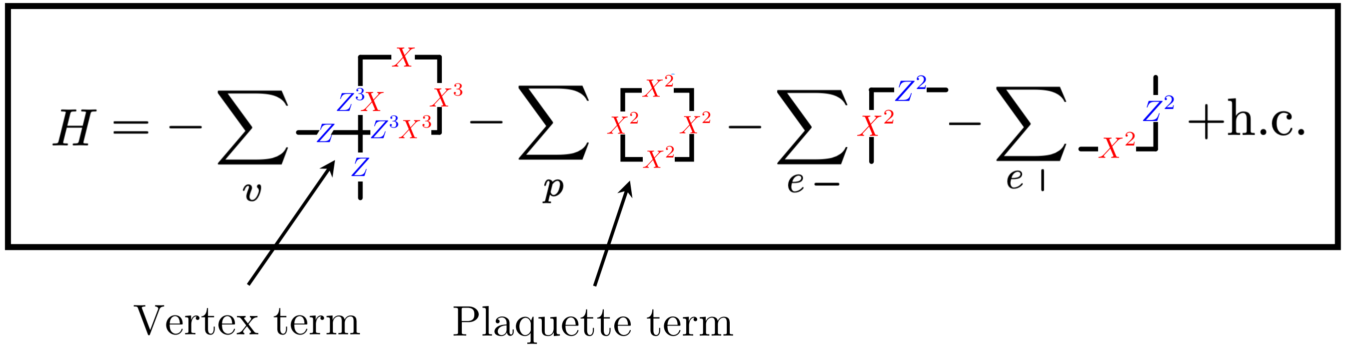
Double semion code on 4D qudits



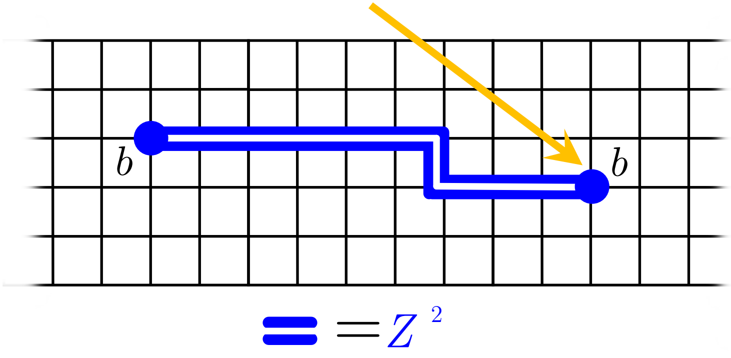
$\circ$ = qubit,

$X^4 = Z^4 = 1$

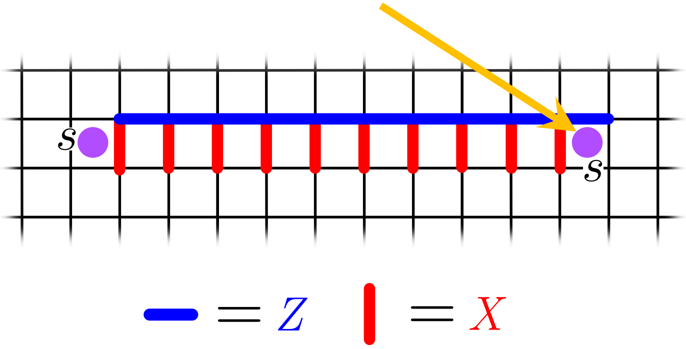
$ZX = iXZ$



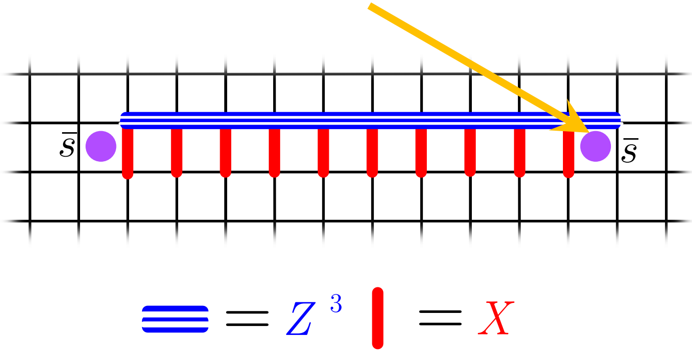
Violates vertex term



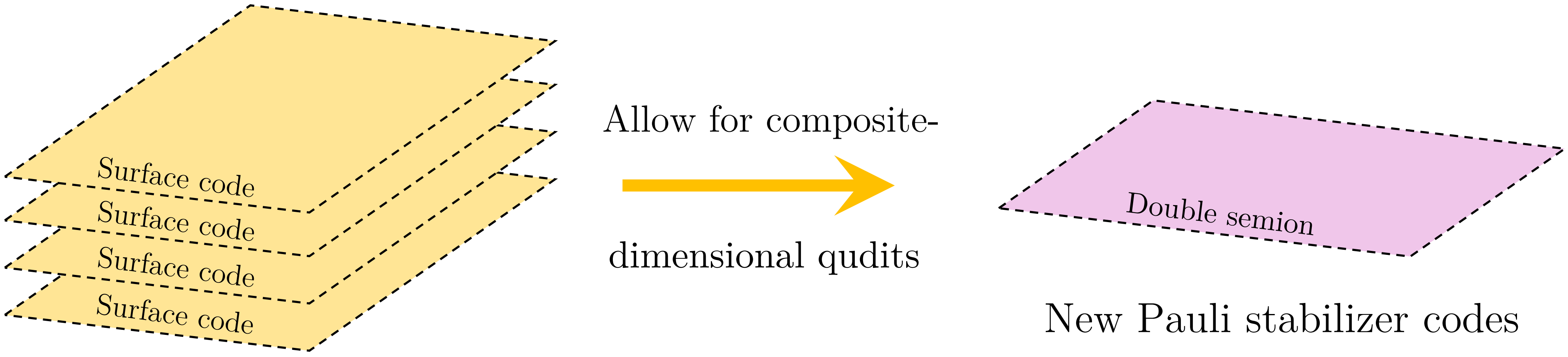
Violates vertex term and plaquette terms



Violates vertex term and plaquette terms



Anyon theories on composite-dimensional qudits



Exhausts all Abelian anyon theories that admit a gapped boundary!

Lagrangian subgroup: Subgroup of bosons $\mathcal{L}$, such that, for every $a \notin \mathcal{L}$, there exists $b \in \mathcal{L}$ with $\theta(ab) \neq \theta(a)\theta(b)$

Which phases admit a stabilizer state?

Trivial phase: ☑

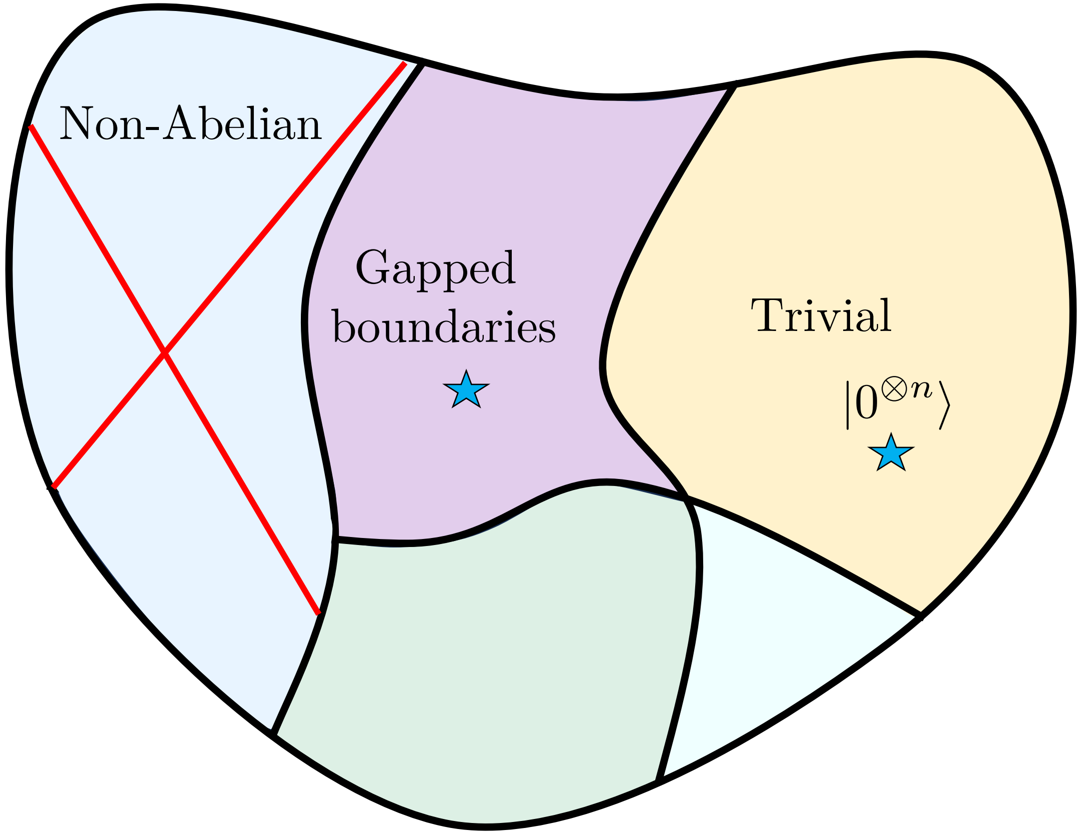
Any computational basis state

Phases with non-Abelian anyons: ☒

Cannot reproduce non-deterministic fusion

Abelian theories with gapped boundaries: ☑

Use composite-dimensional qudits!



Anyon theories without gapped boundaries

Anyon theories without gapped boundaries

→ Non-chiral: $c_- = 0 \mod 8$

→ Chiral: $c_- \neq 0 \mod 8$

Chiral central charge: the chiral central charge c_- is defined mod 8 by

$$e^{2\pi i c_-/8} = \frac{1}{\sqrt{|\mathcal{A}|}} \sum_{a \in \mathcal{A}} \theta(a)$$

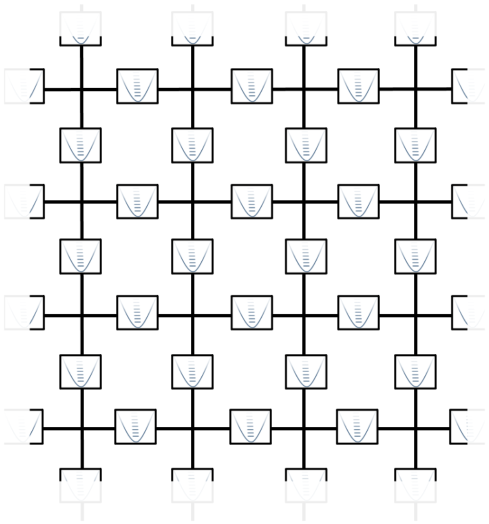
$c_- \neq 0 \mod 8$ indicates that there are gapless chiral edge modes.

Continuous variables

arXiv:2411.04993

Anyon theories on continuous variables

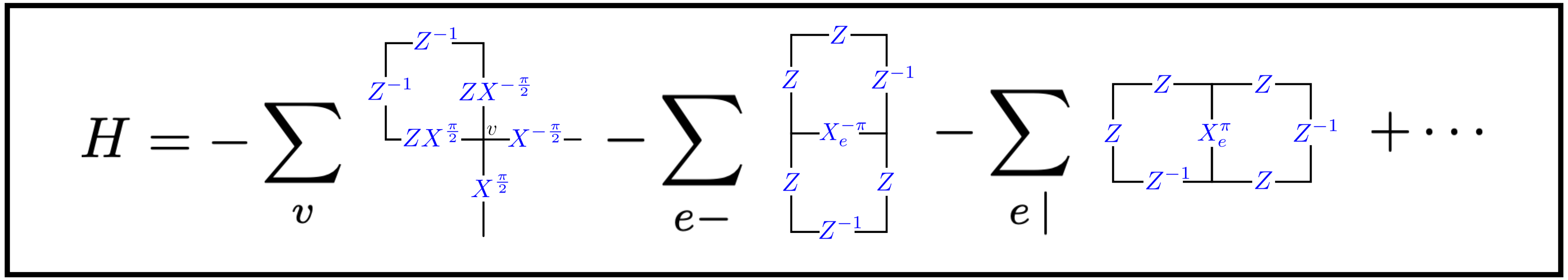
 = CV



<u>Displacement operators:</u> "Pauli operators"

(Heisenberg group)

$$X = e^{-i\hat{p}}, \quad Z = e^{i\hat{x}} \qquad Z^t X^s = e^{ist} X^s Z^t$$



Captures non-chiral Abelian anyon theories <u>without a gapped boundary</u>!

Which phases admit a stabilizer state?

Trivial phase:

Any computational basis state

Phases with non-Abelian anyons:

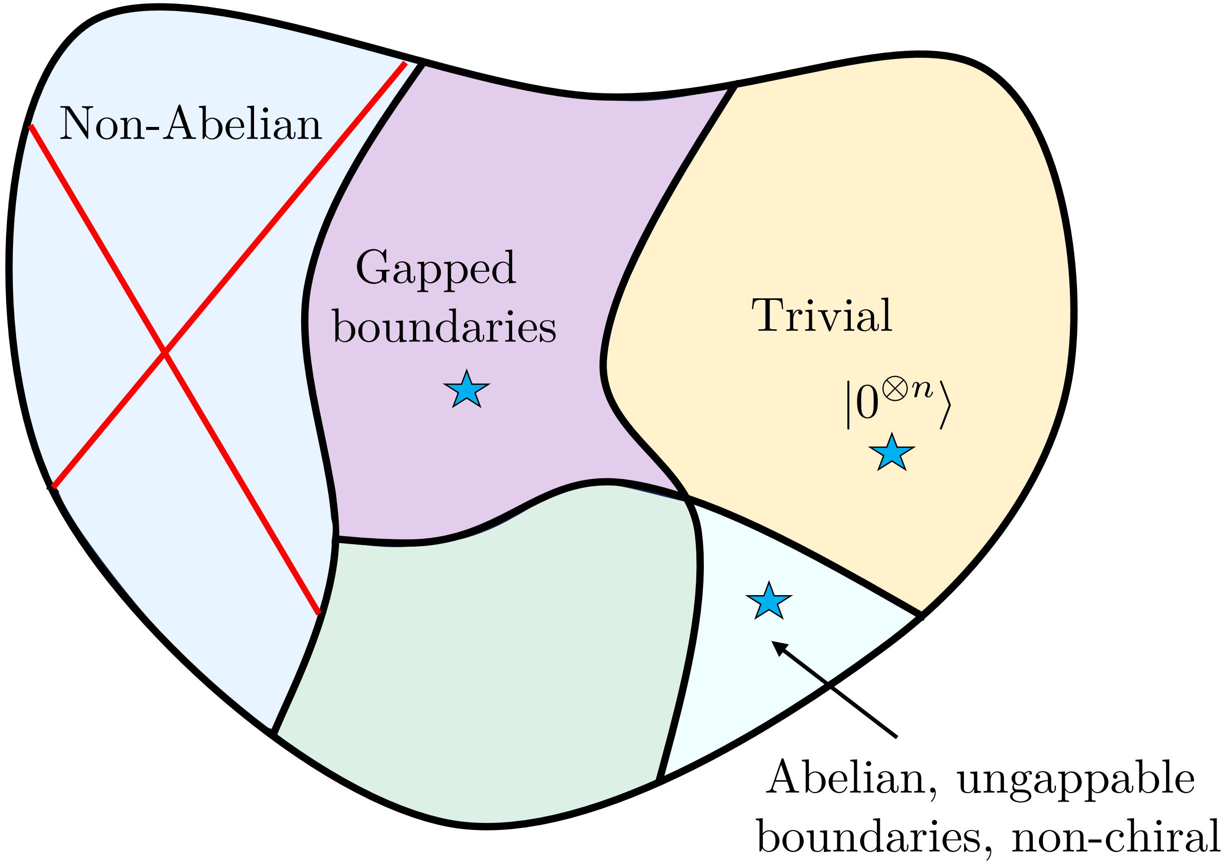
Cannot reproduce non-deterministic fusion

Abelian theories with gapped boundaries:

Use composite-dimensional qudits!

Abelian, ungappable boundaries, non-chiral:

Use continuous variables!



Holographic approach

PRX Quantum **3**, 030326

Chiral semion anyon theory

Anyon types: $\{1, s\}$

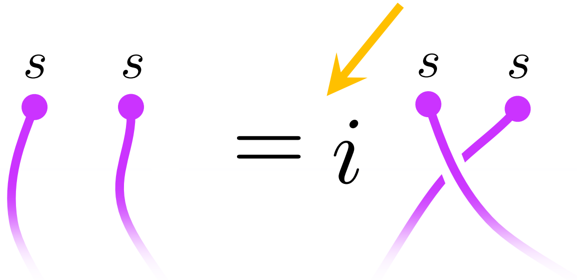
$s =$ 

Fusion rules: $\mathbb{Z}_2$ group generated by s

$$s \times s = 1$$

Exchange statistics:

Semion!



Does not admit gapped boundaries, and is chiral

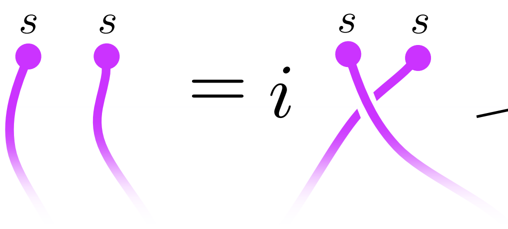
$$e^{2\pi i c_-/8} = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$$

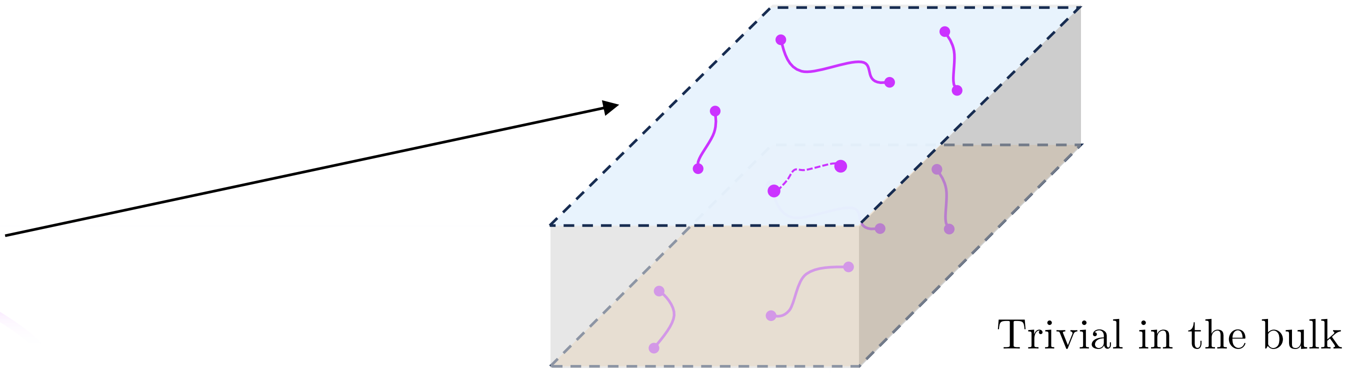
$$\Rightarrow c_- = 1 \text{ mod } 8$$

Anyon theories on the boundary of 3D system

Chiral theories on the boundary of a 3D system?

- Edge modes do not appear!





Captures all Abelian anyon theories!

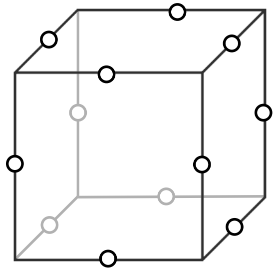
Chiral semion boundary code

Hilbert space:

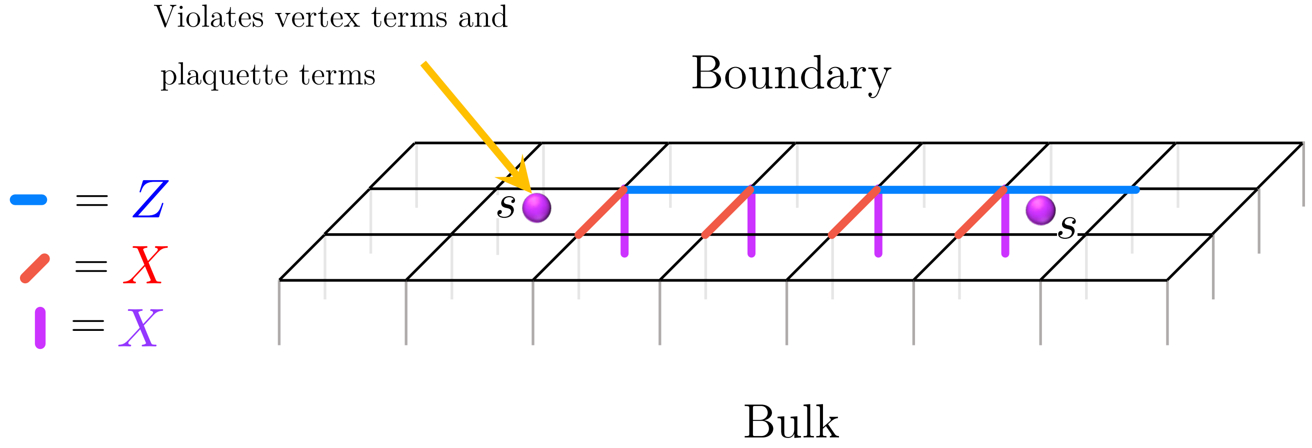
$Z^4 = 1$,

$X^4 = 1$,

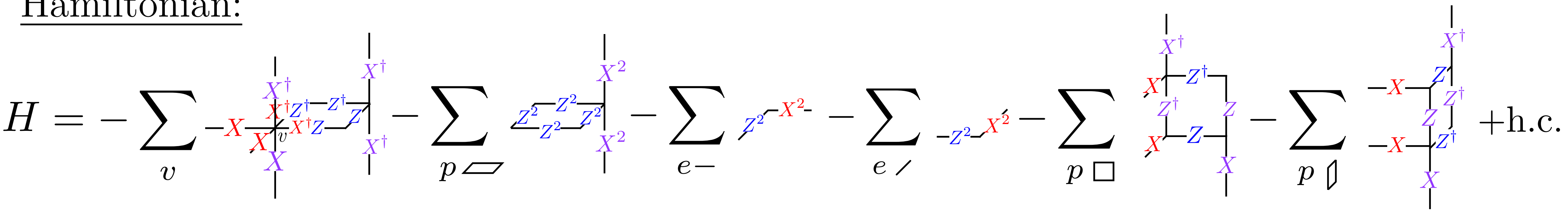
$ZX = i\, XZ$



$\circ$ = 4D qudit



Hamiltonian:



Which phases admit a stabilizer state?

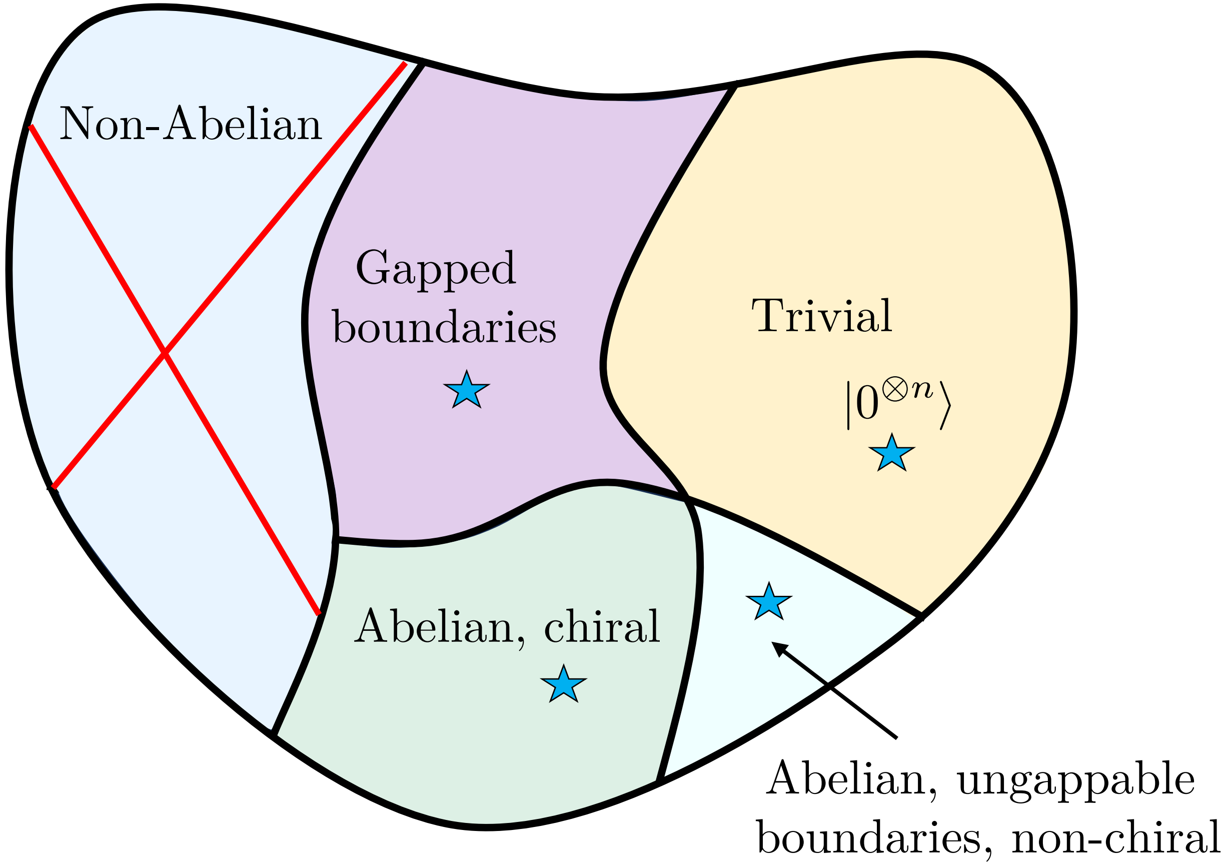
Trivial phase:
Any computational basis state ☑

Phases with non-Abelian anyons:
Cannot reproduce non-deterministic fusion ☒

Abelian theories with gapped boundaries:
Use composite-dimensional qudits! ☑

Abelian, ungappable boundaries, non-chiral:
Use continuous variables! ☑

Abelian, ungappable boundaries, chiral:
Boundaries of 3D! ☑

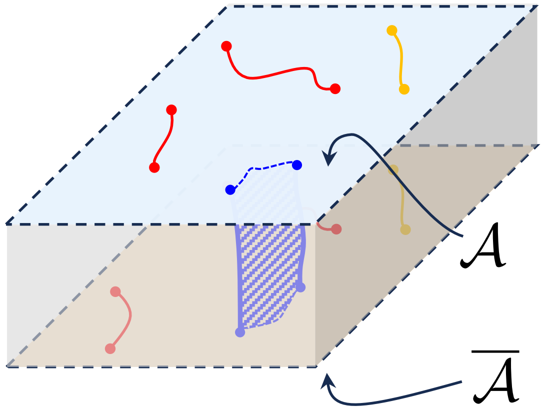


Mixed states

Quantum **7**, 1137
arXiv:2405.02390

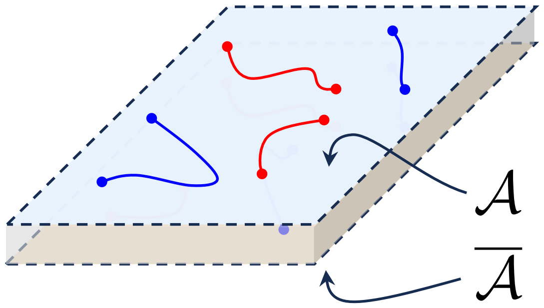
Anyon theories of mixed states

1.



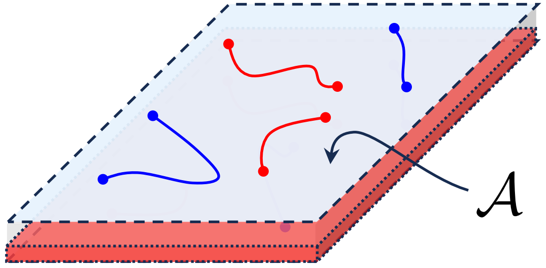
(3+1)D lattice model

2.



Quasi-(2+1)D lattice model

3.



Trace out bottom surface

*Or add fully depolarizing noise

Captures all Abelian anyon theories!

Which phases admit a stabilizer state?

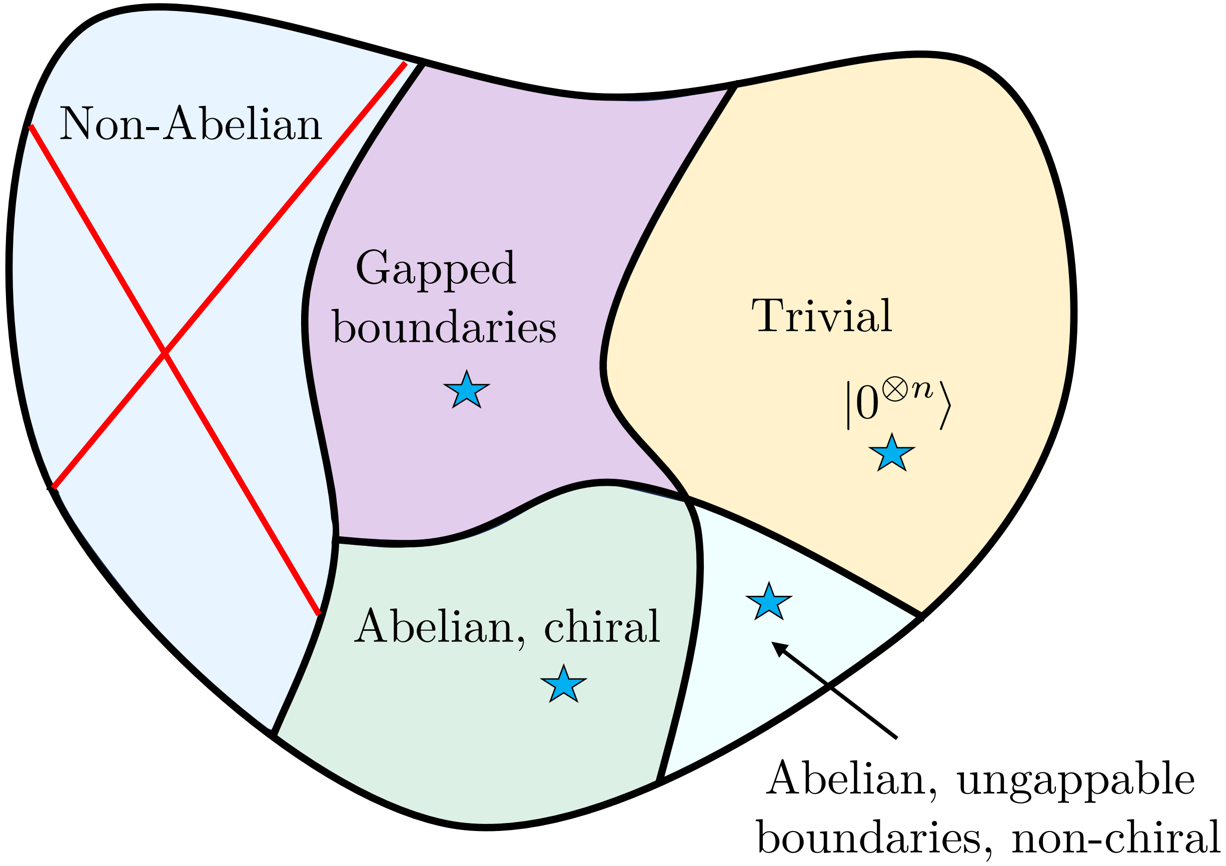
Trivial phase:
Any computational basis state ☑

Phases with non-Abelian anyons:
Cannot reproduce non-deterministic fusion ☒

Abelian theories with gapped boundaries:
Use composite-dimensional qudits! ☑

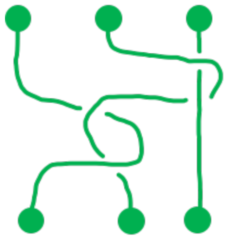
Abelian, ungappable boundaries, non-chiral:
Use continuous variables! ☑

Abelian, ungappable boundaries, chiral:
Boundaries of 3D or mixed states! ☑



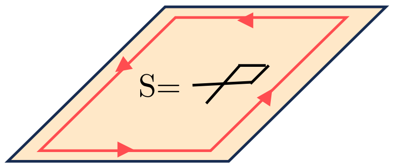
Open questions

Quantifying non-stabilizerness in non-Abelian theories



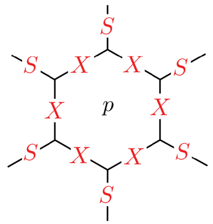
Relation to difficulty of preparation or universality?

Stabilizer models without gapped boundaries on qudits?



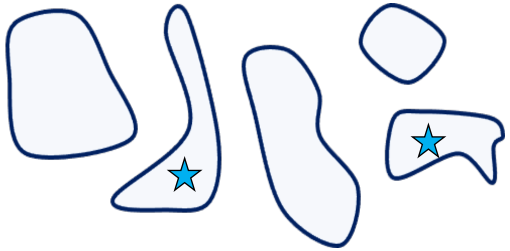
Current folklore: not possible for commuting projector Hamiltonians

Abelian anyon theories on qubits as a source of magic?



Quantify the minimum non-stabilizerness?

Symmetry-enforced magic in 1D systems with SSB?



Fusion-category symmetries enforce magic?

Stabilizer representations of 3D quantum phases of matter?